6.808: Mobile and Sensor Computing
aka IoT Systems

http://6808.github.io

Lecture 7: Introduction to Inertial Sensing & Sensor Fusion

Some material adapted from Gordon Wetzstein (Stanford) and Sam Madden (MIT)

Course Staff	Announcements
Lecturers: Fadel Adib (fadel@mit.edu)	1- PSet 1 due Feb 28
Hari Balakrishnan (hari@csail.mit.edu)	2- Lab 2 due March 2
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Overview: the next two lectures

A deep-dive on **IoT sensing modalities**

this lecture

1. What sensing modalities are relevant for location inference?
2. How can we leverage physics and mathematics fundamentals to build reliable, microscopic sensors?
3. Case-study based application of inertial sensing: Pothole patrol
4. Practical approaches to accounting for sensory noise in real-world settings.

Example Application: Inertial Navigation

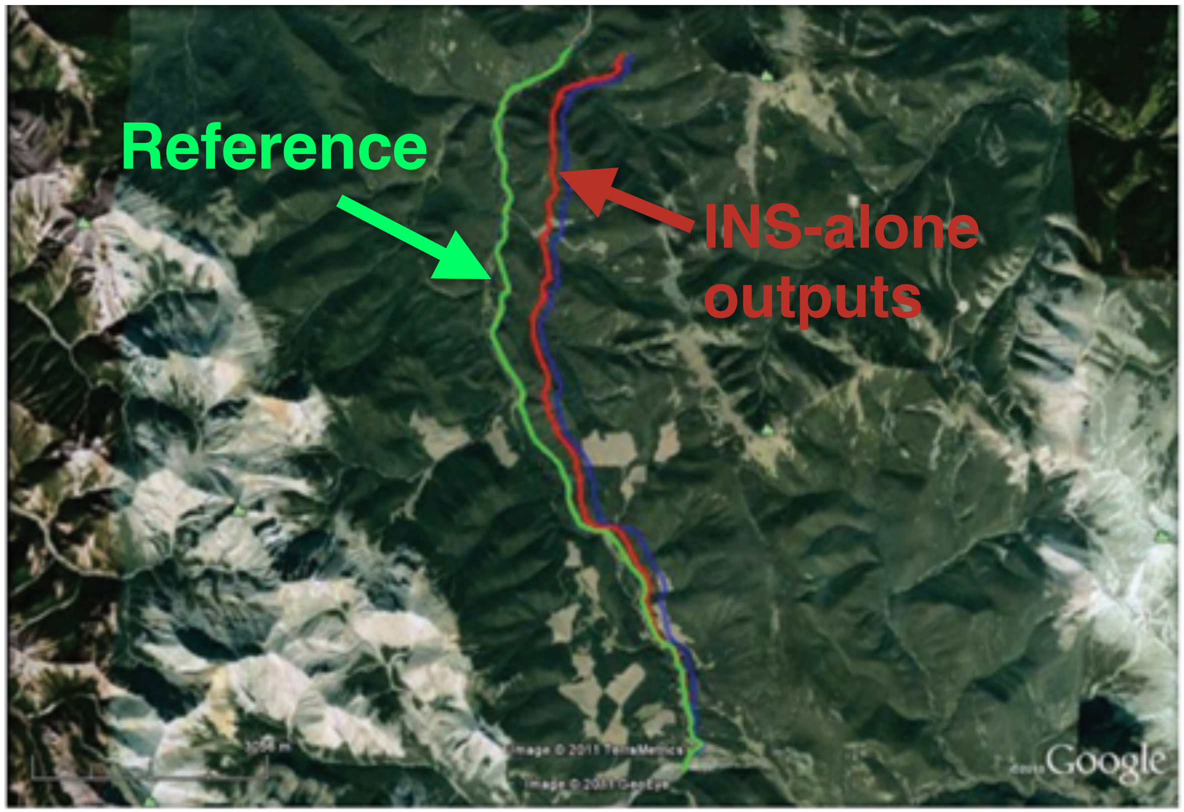
GPS only

GPS+INS

<u>Key Idea #1:</u> Integrate acceleration data over time to discover location (Inertial Sensing)

Inertial Sensing alone is not enough for accurate positioning

- Errors accumulate over time



Source: INS Face Off MEMS versus FOGs

<u>Key Idea #2:</u> Fuse Data from Multiple Sensors (Sensor Fusion)

This Lecture

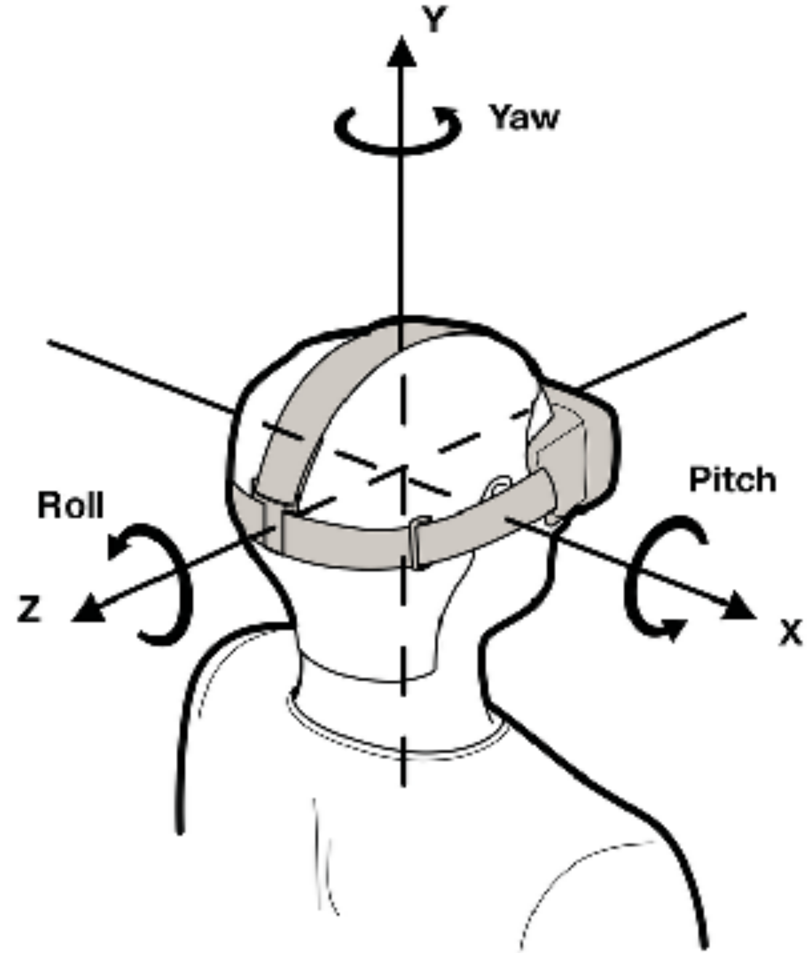
<u>Key Idea #1:</u> Integrate acceleration data over time to discover location (Inertial Sensing)

<u>Key Idea #2:</u> Fuse Data from Multiple Sensors (Sensor Fusion)

Let's understand inertial sensing in the context of VR

- **Goal:** track location and orientation of head or other device

- **Coordinates:** Six degrees of freedom:
  - Cartesian frame of reference (x,y,z)
  - Rotations represented by Euler angles (yaw, pitch roll)



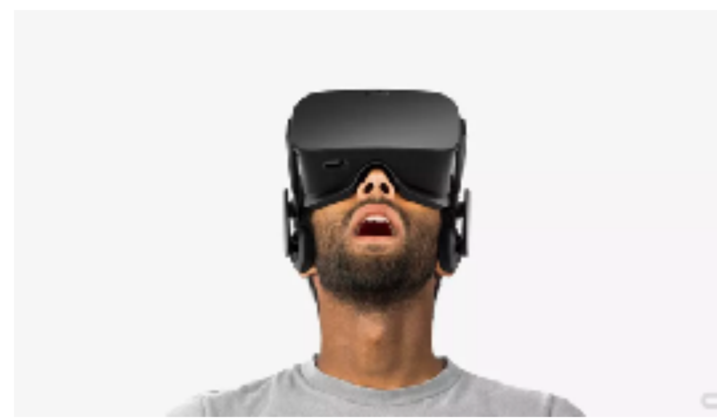
Source: Oculus

What does an IMU consist of? (Inertial Measurement Unit)

- **Gyroscope** measures angular velocity **ω** in degrees/s
- **Accelerometer** measures linear acceleration **a** in m/s^2
- **Magnetometer** measures magnetic field strength **m** in μT (micro-Teslas).

Why is it called IMU?

Where are IMUs used today?



Rest of this Lecture

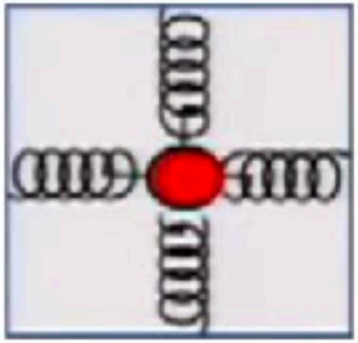
- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer

- Understanding Sources of Errors

- Dead reckoning by fusing multiple sensors

Accelerometer

Mass on spring



Gravity

$1g = 9.8m/s^2$

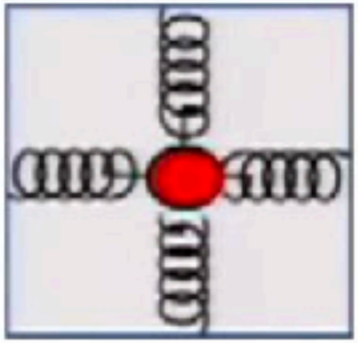
Free Fall

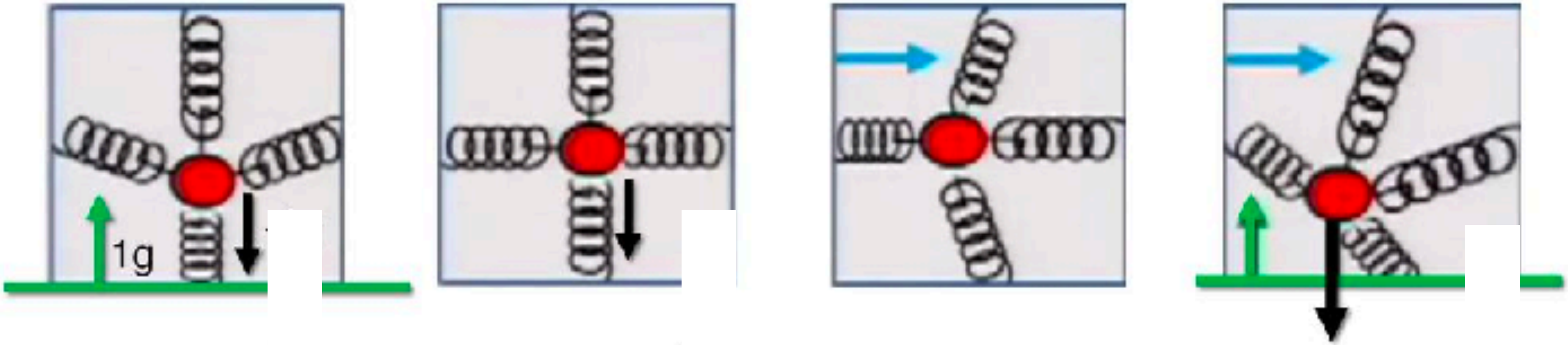
Linear Acceleration

Linear Acceleration plus gravity

Accelerometer

Mass on spring





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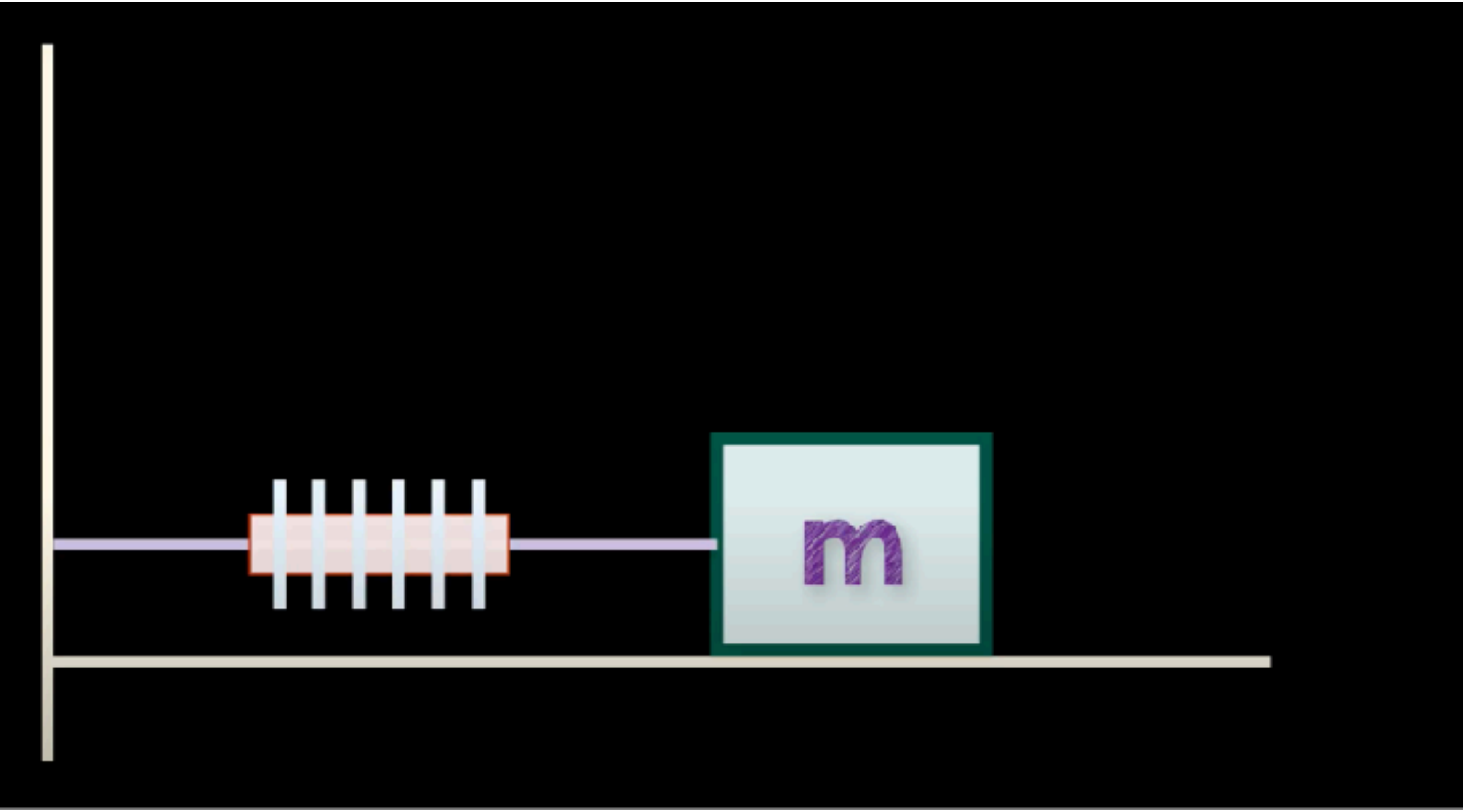
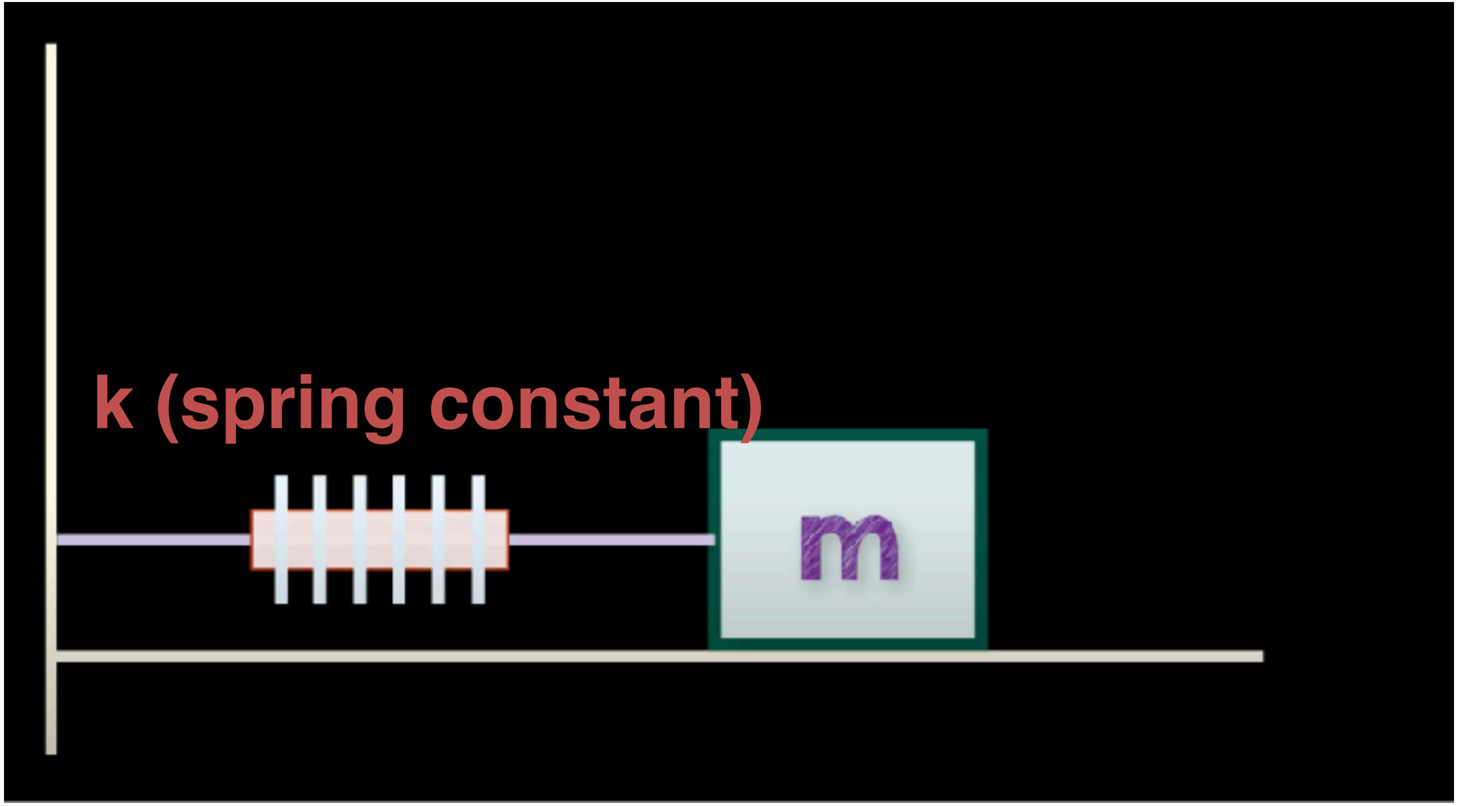
Linear Acceleration

Linear Acceleration plus gravity

How Accelerometers Work



What matters is the displacement



Hooke's Law

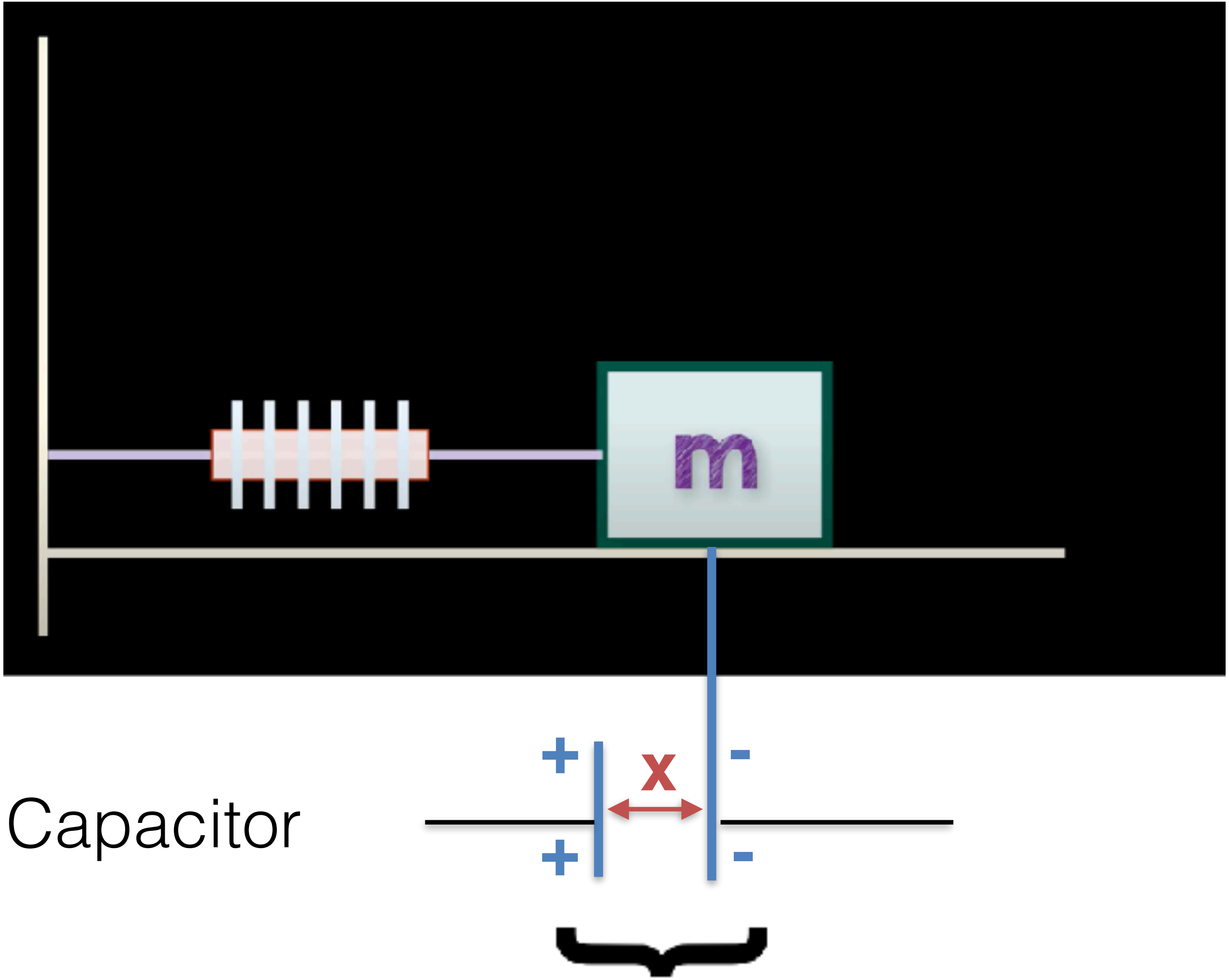
$$F = kx$$

Newton's Law

$$F = ma$$

$$=> a = \frac{k}{m}x$$

Why not simply use displacement to measure displacement?

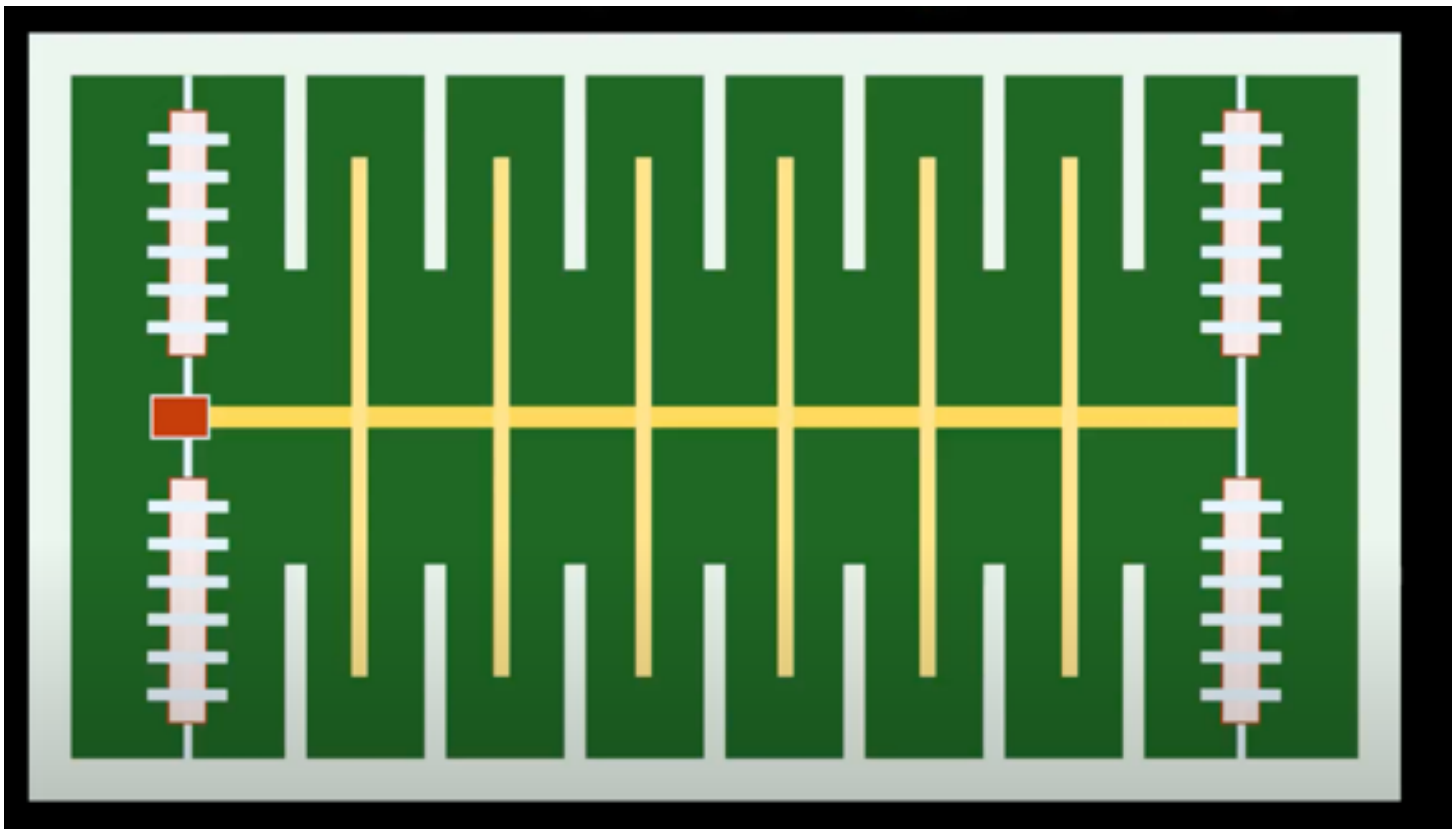


Capacitor

$$C = \epsilon \frac{Area}{x}$$

Measuring Displacement

- How do we measure displacement?
- Most common approach is to use capacitance and MEMS (Micro electro-mechanical systems)



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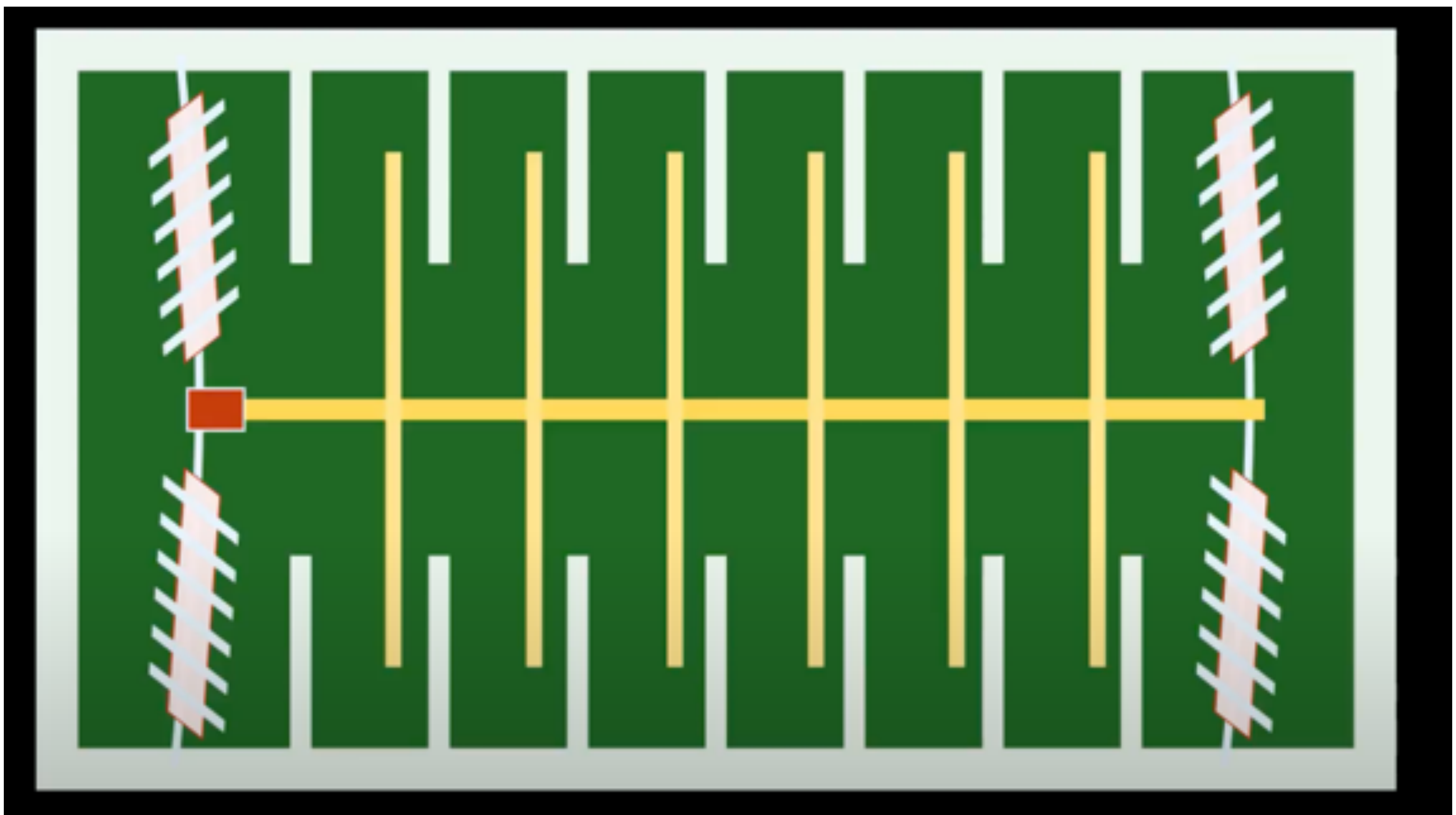
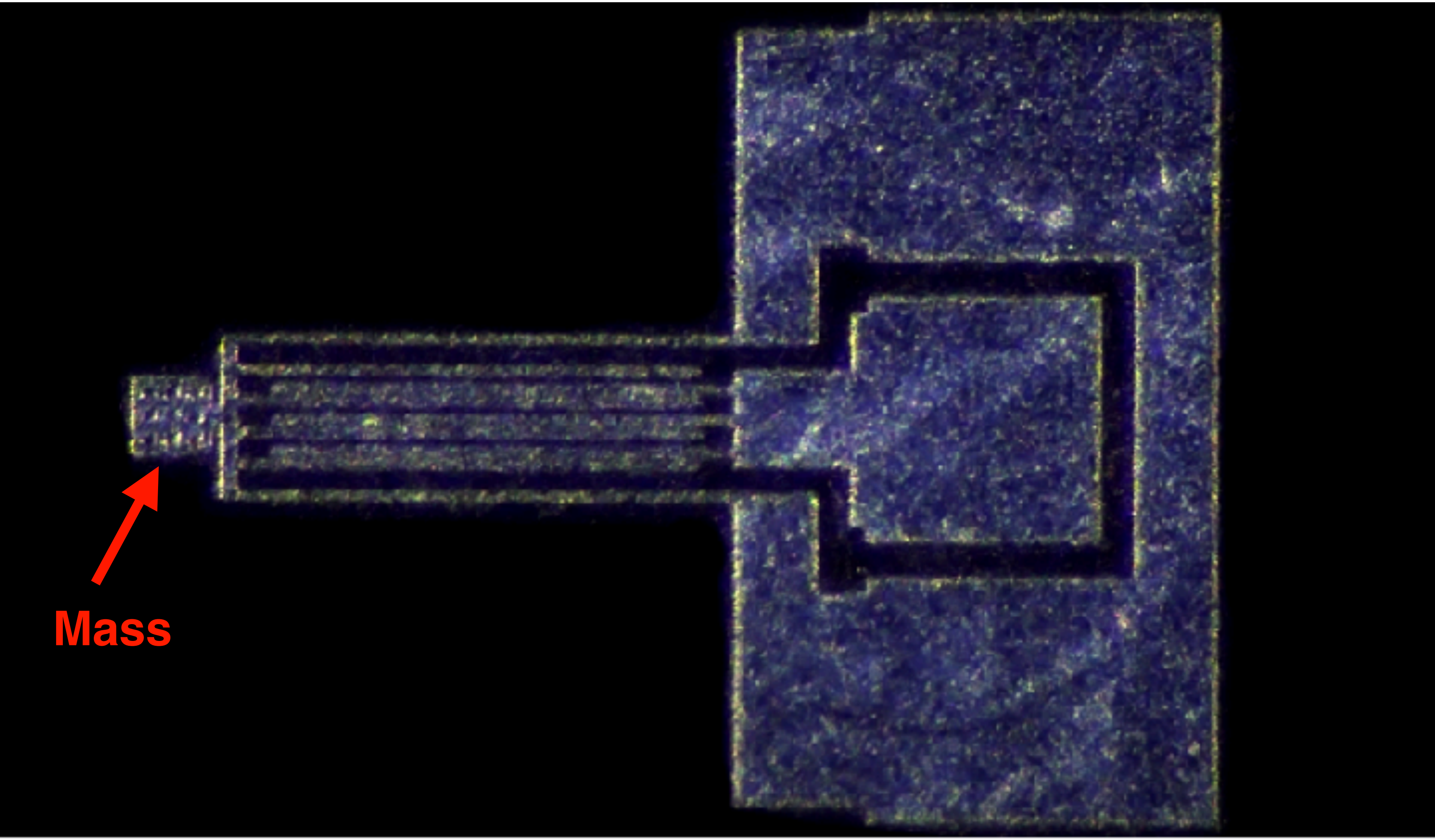
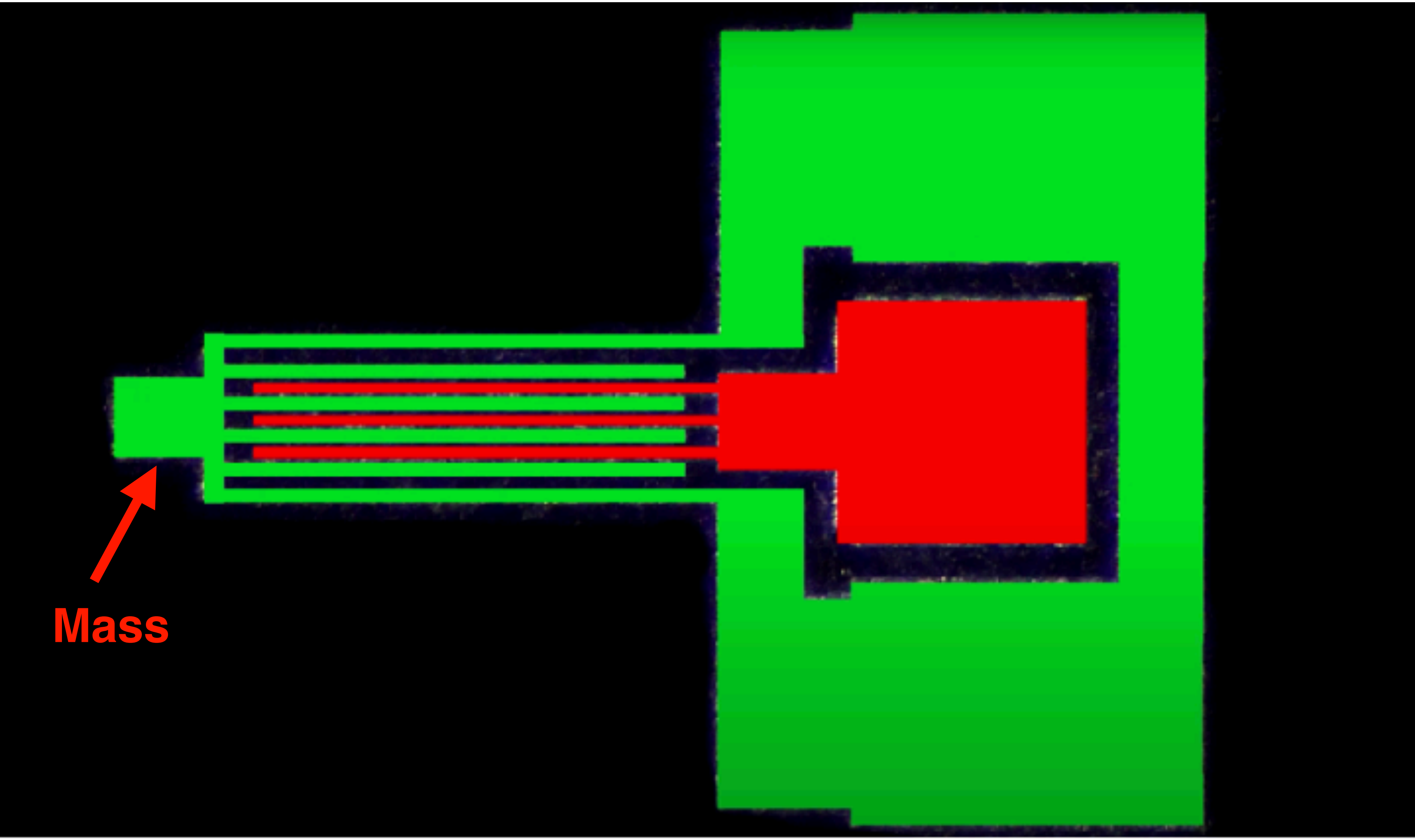


MEMS Accelerometer

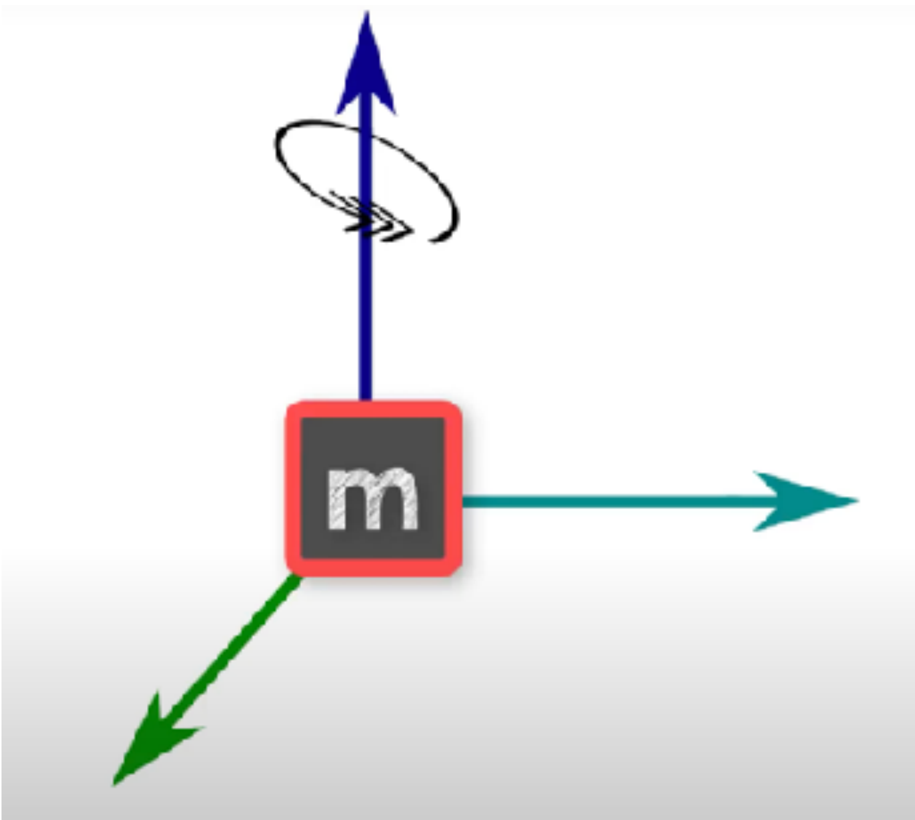


MEMS Accelerometer



How Gyroscopes Work?

The Coriolis Effect

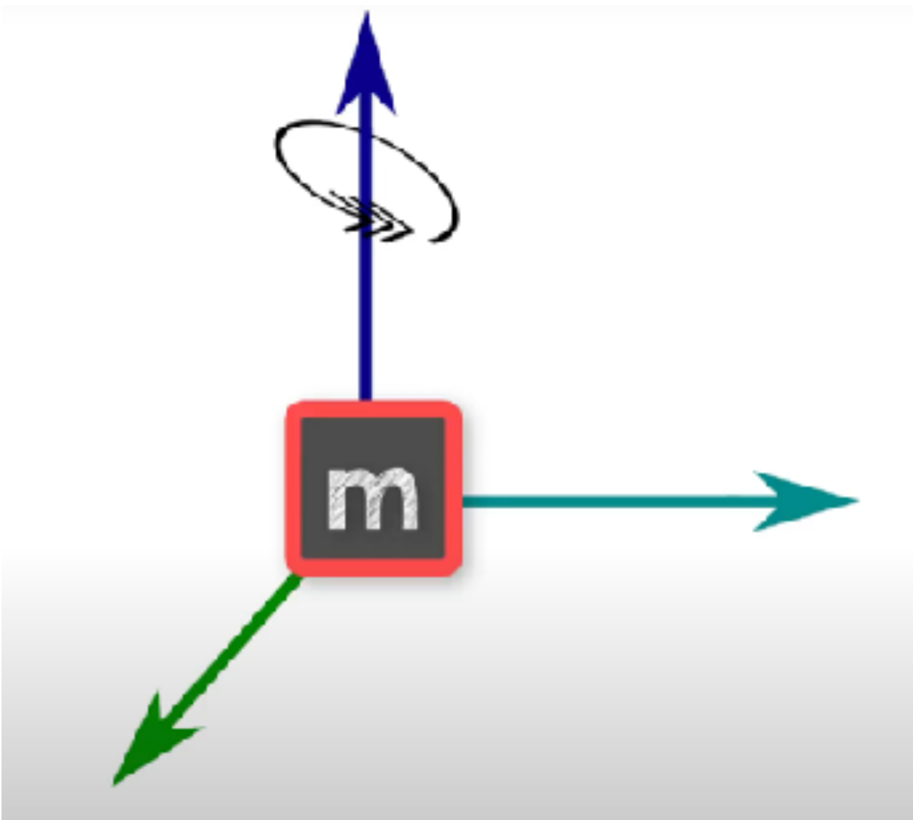


- Assume Vx
- Apply **ω**
- Experiences a fictitious force F(**ω,** Vx) following right hand rule

The Coriolis Effect

How Gyroscopes Work?

The Coriolis Effect

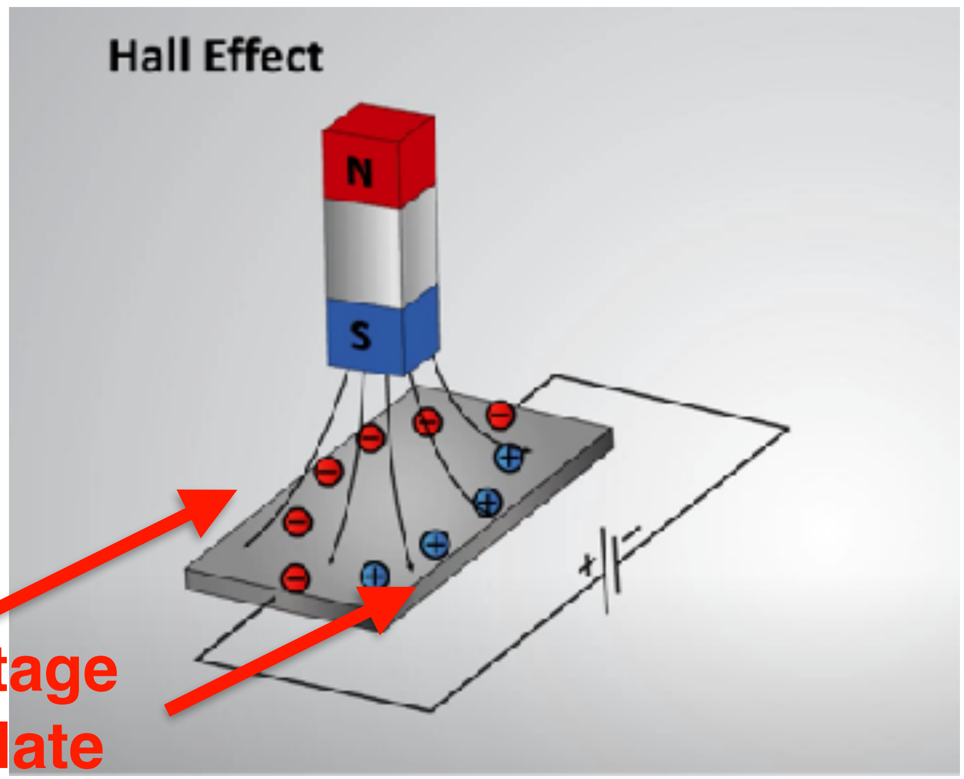


- Assume V_X
- Apply **ω**
- Experiences a fictitious force F(**ω,** Vx) following right hand rule

Can measure F in a similar fashion and use it to recover **ω**

How Magnetometers Work

- E.g., Compass
- Measure Earth’s magnetic field



Measure voltage across the plate

Rest of this Lecture

- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- **Understanding Sources of Errors**
- Dead reckoning by fusing multiple sensors

Gyroscope

Measured angular velocity:

$$\widetilde{\omega} = \omega + b + \eta$$

True angular velocity

Bias

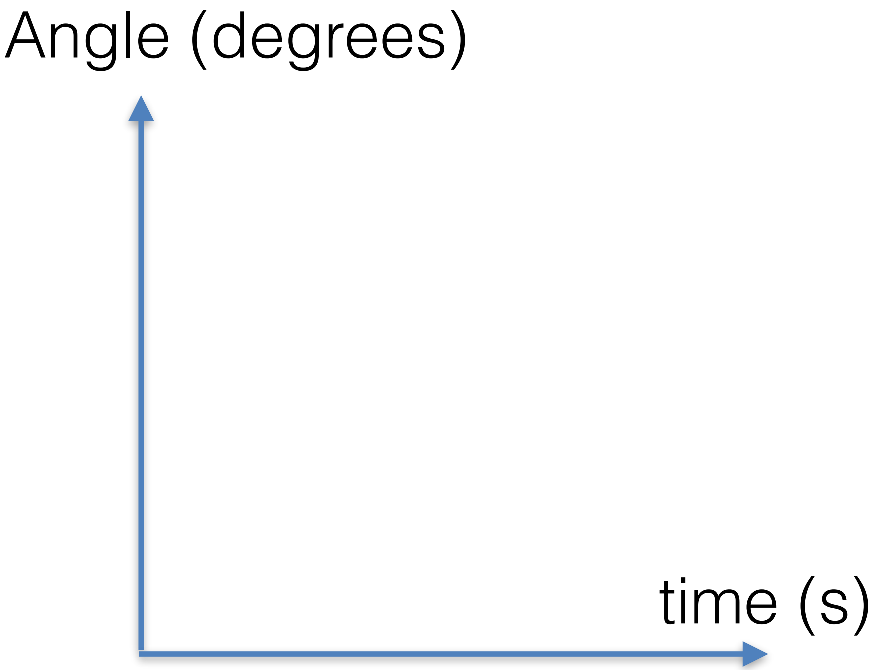
Noise (Gaussian, zero mean)

- How to get from angular velocity to angle?
  - Integrate, knowing initial position

- Linear integration? What are we missing?

Gyroscope- Some Math

Gyro Integration

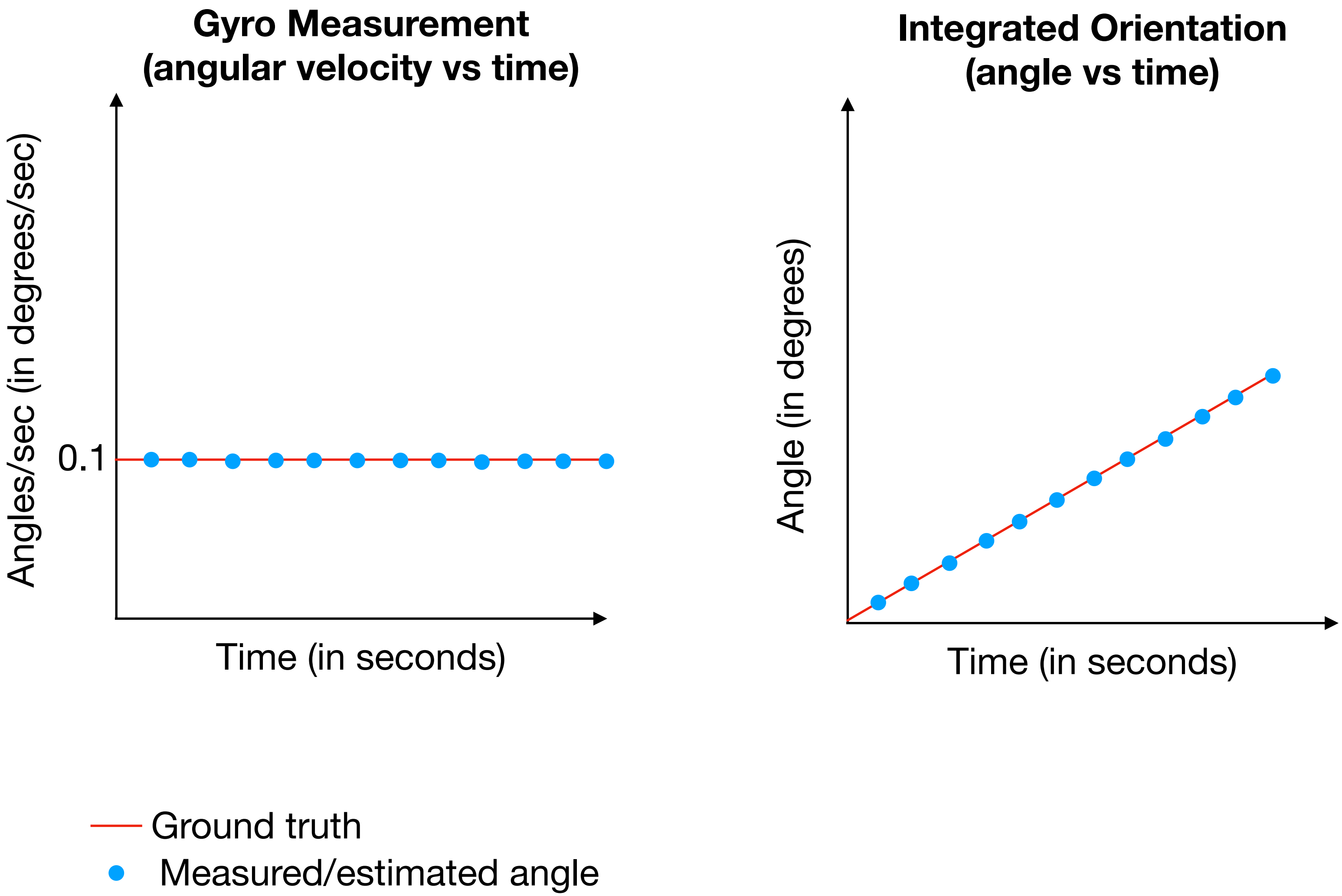


- Let’s plot this for gyro measurement and for orientation
- Let’s include ground truth and measured data for each

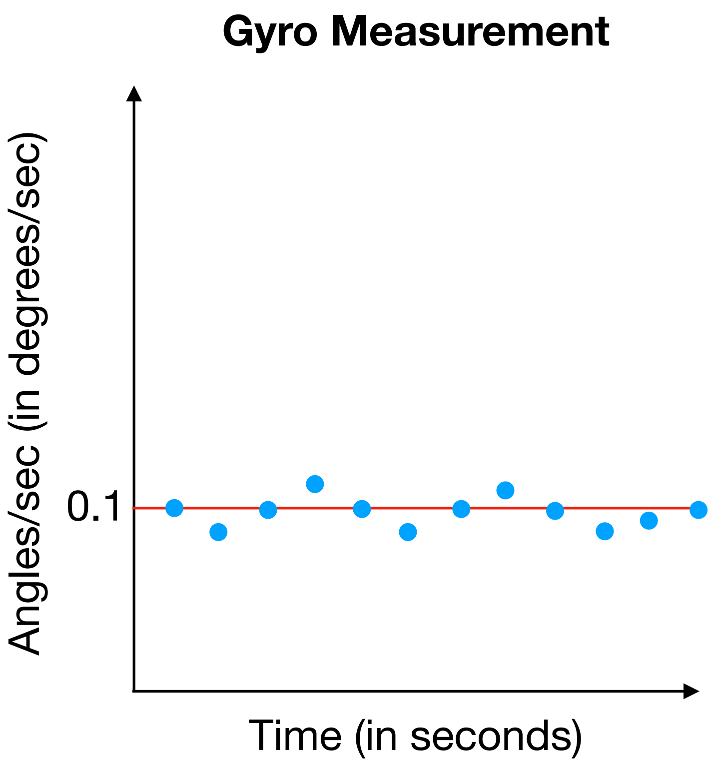
Consider:

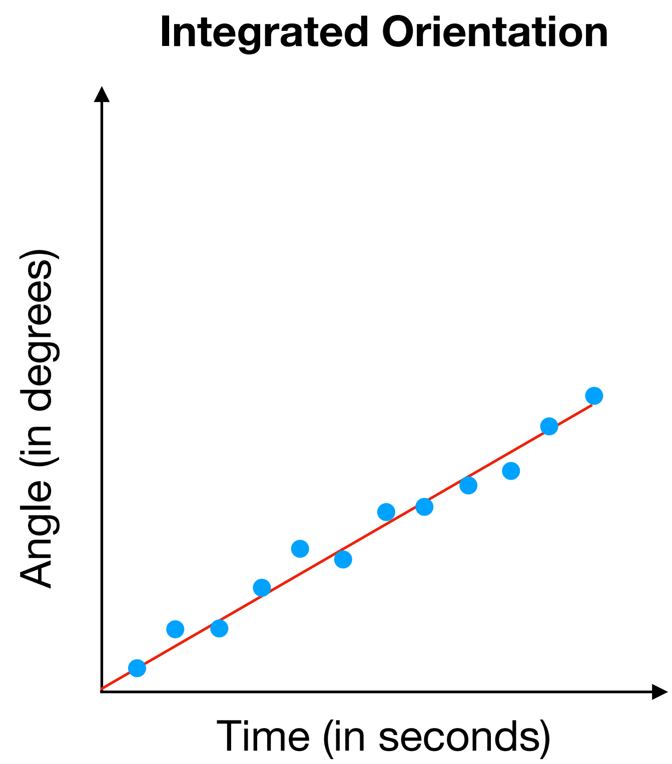
- linear (angular) motion, no noise, no bias
- linear (angular) motion, <u>with</u> noise, no bias
- linear (angular) motion, no noise, bias
- <u>nonlinear</u> motion, no noise, no bias

Gyro integration: linear motion, no noise, no bias

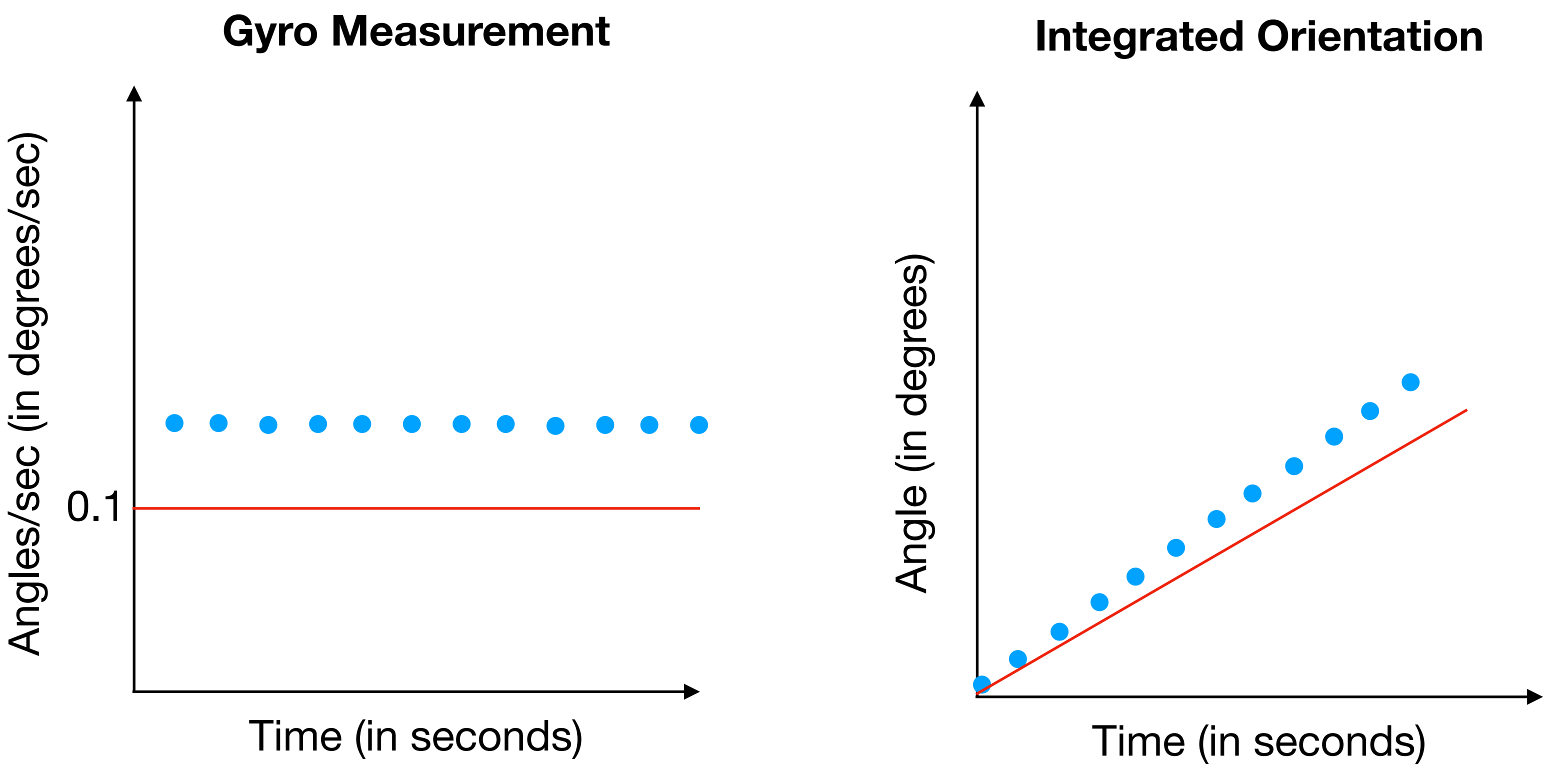


Gyro integration: linear motion, noise, no bias

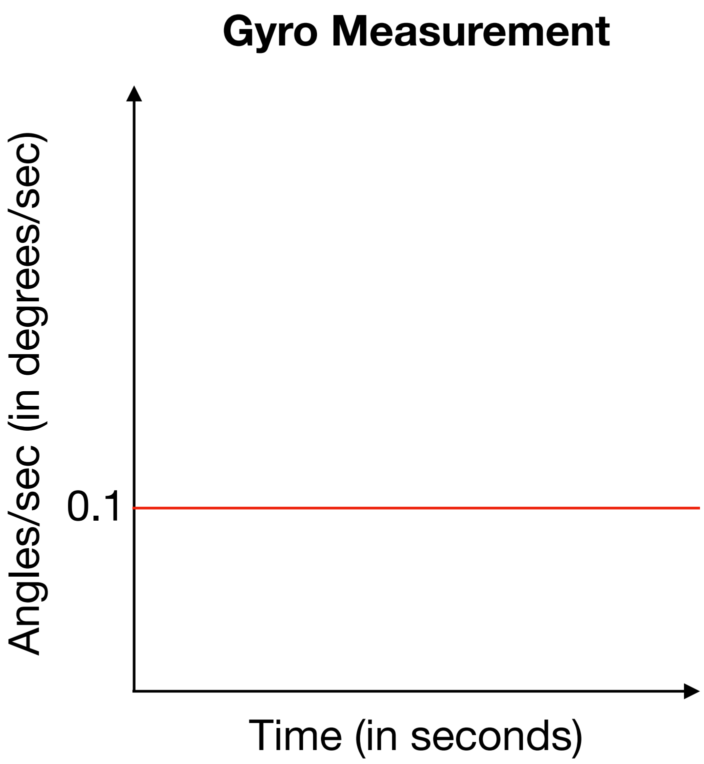


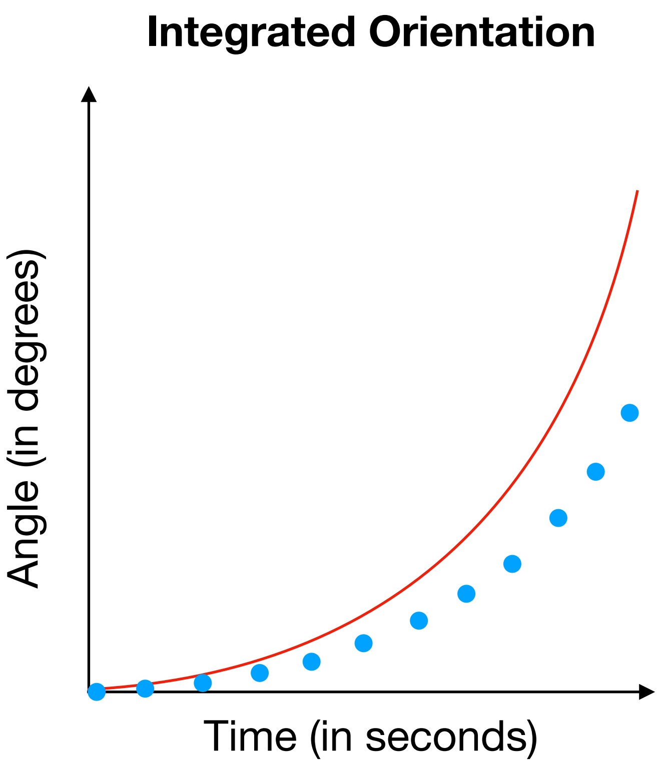


Gyro integration: linear motion, no noise, bias



Gyro integration: nonlinear motion, no noise, no bias





Gyro Integration aka *Dead Reckoning*

- Works well for linear motion, no noise, no bias = unrealistic
- If bias is unknown and noise is zero -> drift (from integration)
- Bias and noise variance can be estimated, other sensor measurements used to correct for drift (sensor fusion)
- Accurate in short term, but not reliable in long term due to drift

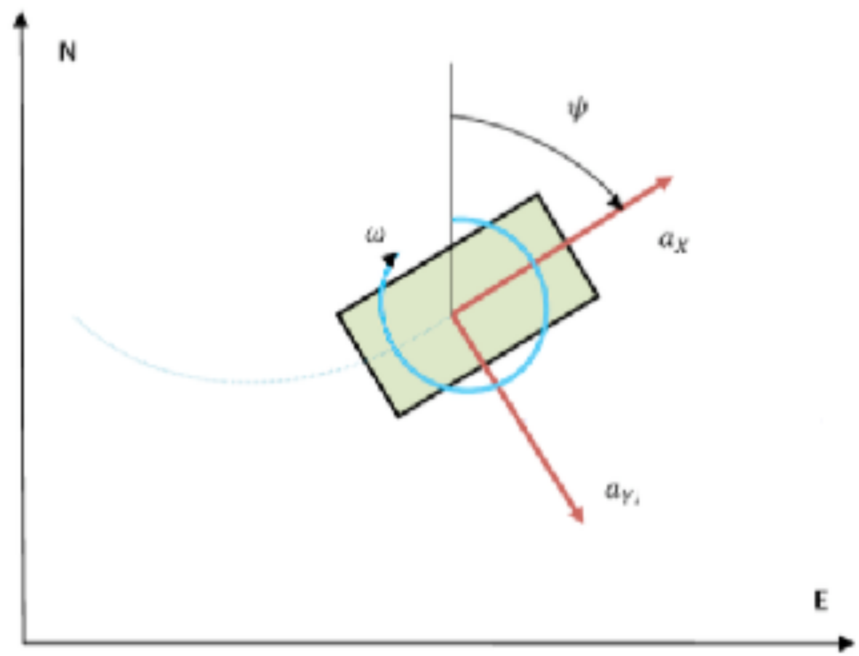
Rest of this Lecture

- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- Understanding Sources of Errors
- **Dead reckoning by fusing multiple sensors**

Dead Reckoning

- The process of calculating one's current position by using a previously determined position, and advancing that position based upon known or estimated speeds over elapsed time and course
- Key things to keep in mind:
  - Frames of reference
  - Orientation change

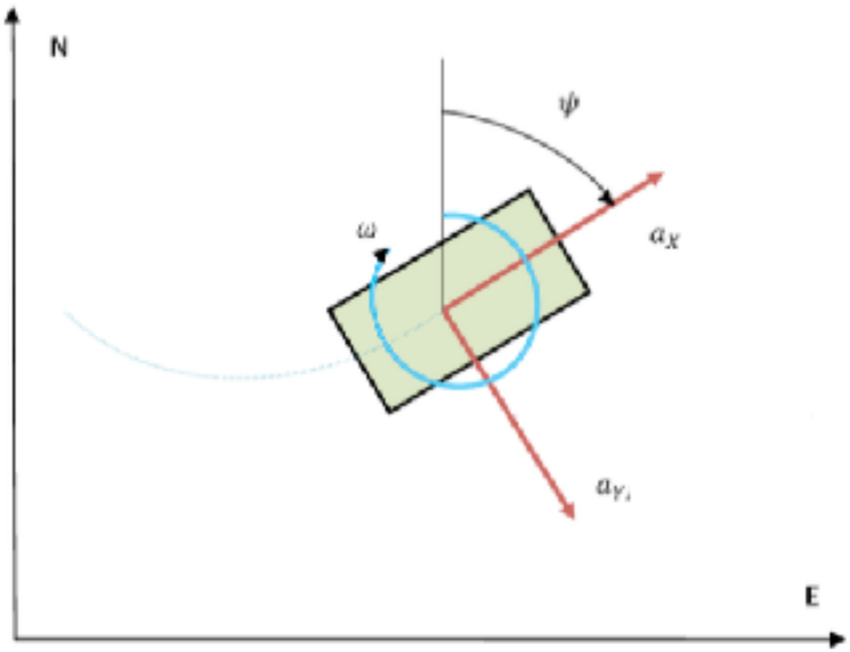
2D Inertial Navigation in Strapdown System



- Have ax, ay, phi; want E, N

Source: Basic Principles of Inertial Navigation
Seminar on inertial navigation systems
Tampere University of Technology

2D Inertial Navigation in Strapdown System



$$\begin{bmatrix} a_N \\ a_E \end{bmatrix} = \begin{bmatrix} cos\psi & -sin\psi \\ sin\psi & cos\psi \end{bmatrix} \begin{bmatrix} a_X \\ a_Y \end{bmatrix}$$

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2D Inertial Navigation in Strapdown System

$$\begin{bmatrix} a_N \\ a_E \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} a_X \\ a_Y \end{bmatrix}$$

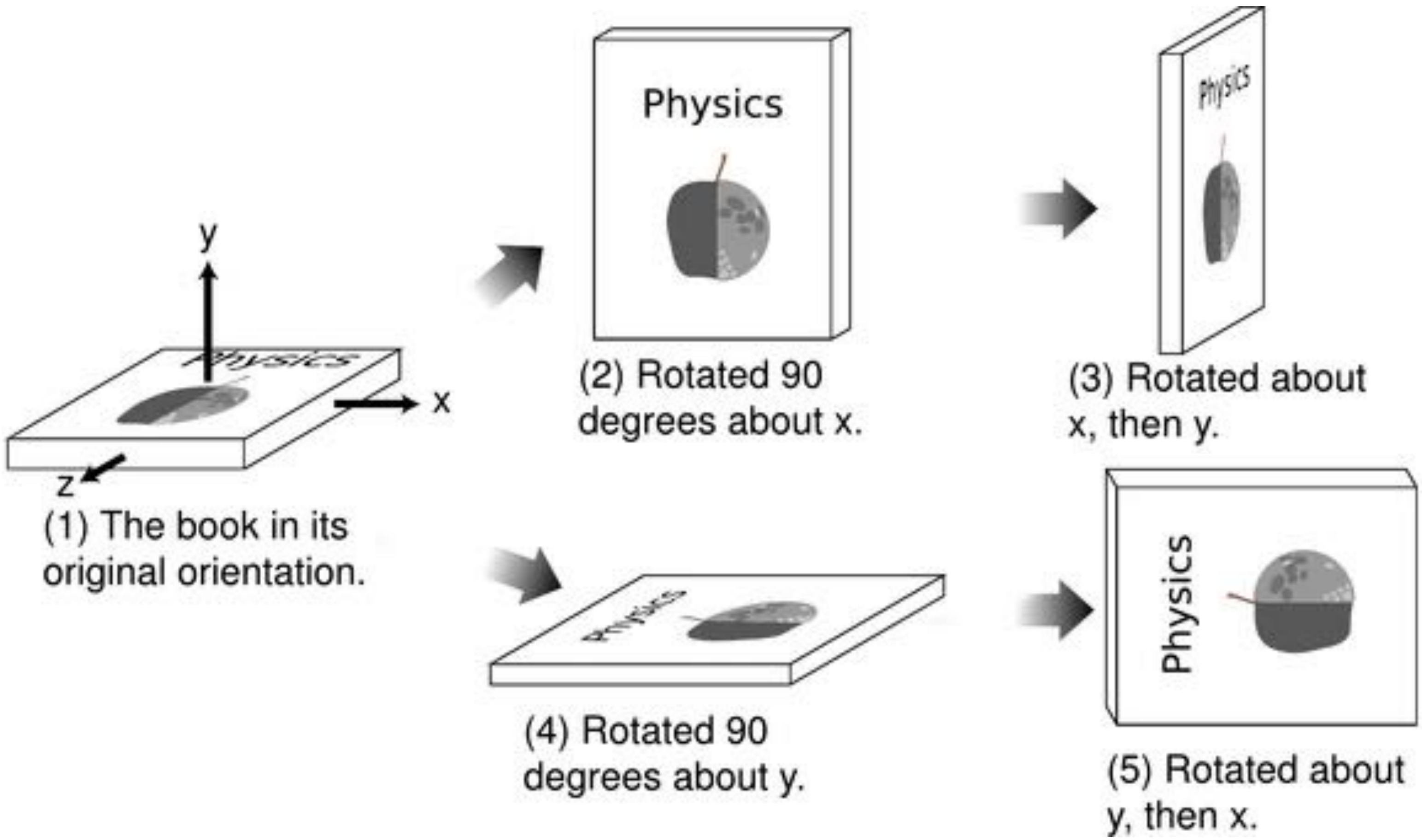
$$V_N(t) = V_N(t_0) + \int_{t_0}^{t} a_N(t)dt$$

$$V_E(t) = V_E(t_0) + \int_{t_0}^{t} a_E(t)dt$$

$$X_N(t) = X_N(t_0) + \int_{t_0}^{t} V_N(t)dt$$

$$X_E(t) = X_E(t_0) + \int_{t_0}^{t} V_E(t)dt$$

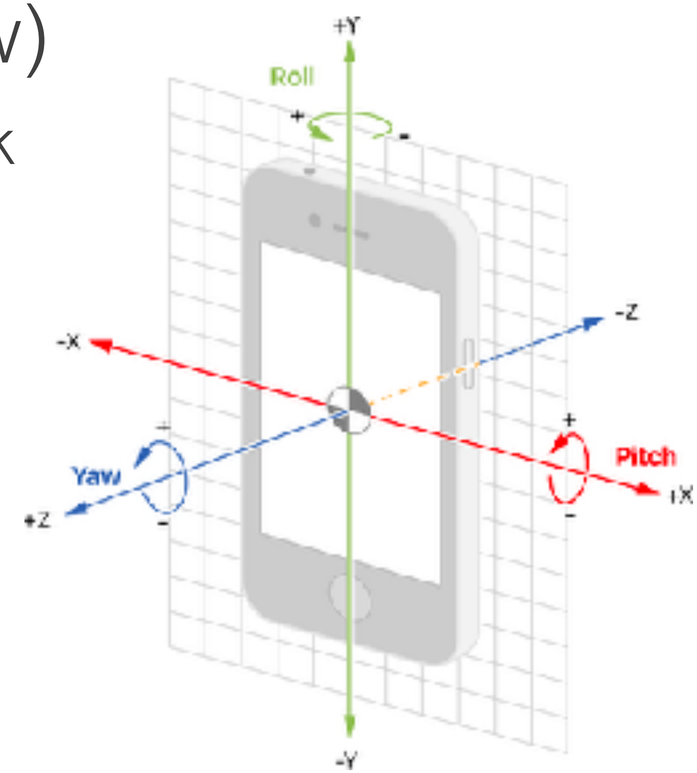
How about 3D Rotations?



Non-commutative = order matters!

3D Rotation Representations

- Rotation Matrix
  - 3 orthonormal vectors = 9 numbers
- Euler Angles (roll, pitch, yaw)
  - Symmetry problem, Gimbal lock
- Axis-angle
- Quaternions



Lecture Recap

- Importance of IMUs for navigation and sensing
- Coordinate systems and 6DOF
- IMU history and current use cases
- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- Understanding Sources of Errors
- Dead reckoning by fusing multiple sensors

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TODO:

1- PSet 1 due Feb 28

2- Lab 2 due March 2